Name of the Course	**: B.A. (Prog)**
Unique Paper Code	**: 62351201_OC**
Name of the Paper	**: Algebra**
Semester	**: II**
Duration	**: 3 Hours**
Maximum Marks	**: 75**

Attempt any four questions. All questions carry equal marks.

1. Consider the vector space $\mathbb{R}^3$ and its subset S

$$S = \{(a,b,c) : 3a - 4b + c = 0, a + 2b - c = 0, \ \ a,b,c \in \mathbb{R}\}$$

Show that S is a subset of $\mathbb{R}^3$ and also find $\dim S$.

Determine whether or not the vectors $(1,-3,2), (2,4,1)$ and $(1,1,1)$ form a basis of $\mathbb{R}^3$

Are the following sets linearly independent or linearly dependent?

$$A = \{(3-i, 2+2i, 4), (2, 2+4i, 3), (1-i, -2i, 1)\}\ ; \quad B = \{(1,2,3), (1,3,2), (3,7,8)\}$$

2. Solve the system of equations:

$$3x + 4y - 6z + w = 7$$
$$x - 2y + 3z - 2w = -1$$
$$x - 3y + 4z - w = -2$$
$$5x - y + z - 2w = 4$$

Find the rank of the matrix

$$A = \begin{pmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

Verify that the matrix B satisfies its own characteristic equation and hence find its inverse

$$B = \begin{pmatrix} 12 & -1 & 4 \\ -1 & 12 & -1 \\ 4 & -1 & 12 \end{pmatrix}.$$

3. Show that

$$2^7 \sin^4 2\theta \sin^4 \theta = \cos 12\theta - 4\cos 10\theta + 2\cos 8\theta + 12\cos 6\theta - 17\cos 4\theta - 8\cos 2\theta + 14$$

Sum the series

$$\sin\theta \sin 2\theta + \sin 2\theta \sin 3\theta + \sin 3\theta \sin 4\theta + \cdots \text{upto n terms}, \qquad n \in \mathbb{N}$$

Solve the equation

$$(z+1)^7 = z^7[\cos 7\alpha + i \sin 7\alpha]\ .$$

4. Solve the equation $\quad x^3 - 9x^2 + 23x - 15 = 0 \quad$; the roots being in A.P.

If α, β, γ are roots of the equation $x^3 - mx^2 + nx - s = 0$, find the value of

$$\sum \alpha^2\beta \quad , \quad \sum \frac{\alpha^2}{\beta\gamma}$$

Find all the roots of the equation $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$, where it is given that $2 + \sqrt{3}$ is one of the roots.

5. Find the multiplicative inverses of the given elements if they exist:

$$[11]_{16} \; in \; \mathbb{Z}_{16} \quad and \quad [38]_{83} \; in \; \mathbb{Z}_{83} .$$

Let a be a fixed element of a group G and let

$$N(t) = \{x \in G : xt = tx\}.$$

Show that $N(t)$ is a subgroup of G.

Find the order and inverse of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 3 & 6 & 1 \end{pmatrix}.$$

6. In permutation group S_6, if $\alpha = (2\;3\;4)$ and $\beta = (2\;5\;6)$, then find a permutation ϕ such that $\phi\alpha\phi^{-1} = \beta$.

Let a be a fixed element of a ring R, and let

$$I_s = \{x \in R : sx = 0\}.$$

Show that I_s is a subring of R.

Show that $\mathbb{Z}_8 = \{0,1,2,3,4,5,6,7,8\}$ is a commutative ring w.r.t. addition modulo 8 and multiplication modulo 8.